Radio Emission from High-Level Transitions in Hydrogen

D. H. Menzel

Phil. Trans. R. Soc. Lond. A 1969 **264**, 249-250
doi: 10.1098/rsta.1969.0020

Radio emission from high-level transitions in hydrogen

By D. H. Menzel

Harvard College Observatory, Cambridge, Massachusetts

Wild (1952) and Kardaschev (1959) noted that atomic transitions between levels of very high quantum number, n, may give lines in the radio region, if the source has sufficiently low density. Gaseous nebulae or H II regions prove to be strong sources of such radiation and a number of the predicted lines have already been found, superposed on a background of continuous emission.

Relativistic effects prove to be negligible. Doppler broadening must eventually include macroscopic motions as well as temperature effects. H. R. Griem (1966, unpublished) has shown that a proper treatment of ion and electron collisions replaces the linear Stark effect. However, a broadening still remains as the result of departure of the potential of the atom from a Coulomb field because of surrounding ions. The theory follows lines similar to those laid down by Debye and Hückel in their study of dilute electrolytes.

A full wave-mechanical solution leads to a simple, exact expression for the values of $f_{n'n}$ for high-level transitions. Let $n-n' = c$, where c is a positive integer, small compared with n or n'. Then we can write

$$f_{n'n} = [4n' J_c(c)\, J_c'(c)]/3c^2 = n' M(c), \tag{1}$$

where $J_c(c)$ is a Bessel function of equal order and argument. The prime denotes differentiation with respect to the argument. The function $M(c)$ has been tabulated.

These f-values obey the sum rule exactly,

$$\sum_n f_{n'n} = 1. \tag{2}$$

For this summation n' is constant and n must range over all quantum numbers. When $n < n'$, however, we must write

$$f_{n'n} = -\frac{n^2}{n'^2} f_{nn'}. \tag{3}$$

Note the negative value. During the summation these negatives cancel much of the positive contribution leaving the sum

$$3 \sum_1^\infty cM(c) = 1 \tag{4}$$

as a new relation involving Bessel functions, which can be derived independently. This derivation resolves the question, first raised by Kardaschev (1959) of reconciling f-values larger than unity with the f sum rule.

The emission lines are superposed on a background continuum, resulting from three main causes: free-free emission, free-bound emission involving high quantum numbers, and broadened lines from still higher transitions. All of these processes have been evaluated quantitatively to yield the equation of radiative transfer in continuum and line. A revaluation of the mean Gaunt factor for the free-free emission gave the formula

$$\bar{g} = \frac{\sqrt{3}}{\pi}[\ln(4kT/h\nu) - \gamma], \tag{5}$$

where γ is Euler's constant, 0·5772157. This formula is preferable to the one commonly employed by radio astronomers.

The equations of radiative transfer must properly allow, as Goldberg (1966) first showed, for the effect of stimulated emission and departures from thermodynamic equilibrium. The study leads finally to prediction of the total emission from an H II region.

This work was supported in part by the National Science Foundation, under Contract NSF GP 5791 with Harvard University.

REFERENCES (Menzel)

Goldberg, L. 1966 *Astrophys. J.* **144**, 1225.
Kardaschev, N. S. 1959 *Astronom. Zhu.* **36**, 838.